Berkeley CS276 & MIT 6.875

Specialized homomorphic encryption, commitments and applications

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Announcements

- Starting to record

Specialized/partial homomorphic encryption

- An encryption scheme that is homomorphic with respect to a specific function, and cannot compute arbitrary functions like FHE
- Usually faster than FHE due to specialization (but not always)

El Gamal encryption (1985)

A semantically secure public-key encryption scheme

Setup(1^k):
- Generate large prime p of size k
- Choose generator $1 < g < p-1$
- Output (p, g)

KeyGen(1^k):
- Choose random $0 \leq \text{sk} \leq p-2$
- Let $pk = g^{sk} \ mod \ p$
- Output (sk, pk)

Enc(pk, m): $m \in [1, p-1]$ Why?
- Choose random $0 \leq r \leq p-2$
- Output $(g^r mod\ p, m \times pk^r \ mod\ p)$

Dec$\big(sk, (c_1, c_2)\big)$: How to decrypt?
- Output $c_2 c_1^{-sk} \ mod\ p$

$$c_2 c_1^{-sk} = m\, pk^r \ g^{-rsk} = m\ g^{sk\, r} g^{-r\, sk} = m$$

DDH assumption

Enc(pk, m):

- Choose random $0 \le r \le p-2$
- Output $(g^r mod\ p, m \times pk^r\ mod\ p)$

Diffie-Hellman key exchange in disguise + used as one time pad

Semantic security relies on the Decisional Diffie Hellman assumption:
For all nonuniform PPT A,

$$|\Pr[(g,p) \leftarrow Setup(1^k); a,b \leftarrow [0,p-2], A(p,g,g^a,g^b,\boldsymbol{g^{ab}}) = 1] -$$
$$\Pr[(g,p) \leftarrow Setup(1^k); a,b,c \leftarrow [0,p-2], A(p,g,g^a,g^b,\boldsymbol{g^c}) = 1]| < negl(k)$$

Proof of security

Decisional Diffie Hellman assumption: ∀ nonuniform PPT A,

$$| \Pr\left[(g,p) \leftarrow Setup(1^k); a,b \leftarrow [0,p-2], A\left(p,g,g^a,g^b,\boldsymbol{g^{ab}}\right) = 1\right] -$$
$$\Pr\left[(g,p) \leftarrow Setup(1^k); a,b,c \leftarrow [0,p-2], A\left(p,g,g^a,g^b,\boldsymbol{g^{c}}\right) = 1\right] | < negl(k)$$

Claim: If DDH holds, El Gamal is semantically secure.

Proof: Assume A can break El Gamal's security, let's show that B can break DDH.

B must distinguish between g^a, g^b, g^{ab} and g^a, g^b, g^c

A can distinguish between $g^{sk}, g^r, m_0\, g^{skr}$ and $g^{sk}, g^r, m_1 g^{sk\, r}$

B feeds g^{ab} or g^c times m_b to A for b random. If it is g^c, A cannot guess, else A guesses correctly.

Other partially homomorphic encryption schemes

Scheme	Homomorphism
Goldwasser-Micali'82	XOR
Paillier'99	+
Boneh-Goh-Nissim'05	+, then one *, then + based on bilinear maps
PHE/SHE (partially homomorphic encryption)	Some polynomial

Recall: commitments

A commitment protocol is an efficient two-stage protocol between a sender S and a receiver R:

- commitment stage: S has private input *x*. At the end of the stage,
  - Both parties hold *com* (commitment)
  - S holds *r* (the randomness used for decommitment)
- reveal stage: S sends *(r, x)* to R, which accepts or rejects

Completeness: R always accepts in an honest execution of S.

Hiding: Hiding:∀ R*, $x \neq x'$, in commit stage

$$\{ \text{View}(S(x),R^*)(1^k) \} \approx_c \{ \text{View}(S(x'),R^*)(1^k) \}.$$

Binding: Let *com* be output of commit stage, ∀S*

Prob[S* can reveal two pairs (r,x) & (r',x') s.t. R(com, r, x) = R(com, r′, x′) = Accept] <negl(k)

Pedersen commitment

Setup (1^k) - at the receiver:

- select large primes p and q of size k such that q divides $p-1$
- select a generator g of the order-q subgroup of Z_p^*
- generate randomly $a \leftarrow Z_q$
- let $h = g^a \; mod \; p$
- output (g, h, p)

Commit(g, h, p, x) - by the sender:

- choose random $r \leftarrow Z_q$
- output $comm = g^x h^r \; mod \; p$

Reveal - by the sender:

- send x and r to receiver
- the receiver verifies that $comm = g^x h^r \; mod \; p$ and accepts if so, else rejects

Perfectly hiding

Commit(g, h, p, x) - by the sender:

- choose random $r \leftarrow Z_q$
- output $comm = g^x h^r \ mod\ p$

- For a commitment $comm$, every x could have been committed to in $comm$
- Given x, r and any x', $\exists r'$ such that $g^x h^r = g^{x'} h^{r'}$

$$r' = (x - x')a^{-1} + r \mod q$$

Computationally binding

- Assume the sender can find x', r', s.t $x' \neq x$ and

$$comm = g^x\ h^r = g^{x'} h^{r'}$$

- $h = g^a \, mod \, p$ implies $x + ar \; = \; x' + ar' \, mod \, q$
- The sender can compute $a \; = \; (x' - x)(r - r')^{-1}$

=> Sender solved discrete logarithm of h base g!!

Why is Pedersen homomorphic?

Commit(g, h, p, x) - by the sender:

- choose random $r \leftarrow Z_q$
- output $comm(x, r) = g^x h^r \; mod \; p$

$$comm(x_1, r_1) * comm(x_2, r_2) = g^{x_1 + x_2} h^{r_1 + r_2} \; mod \; p$$

The sender reveals this commitment by showing $x_1 + x_2$ and $r_1 + r_2$

[Narula-Wasquez-Virza'18]

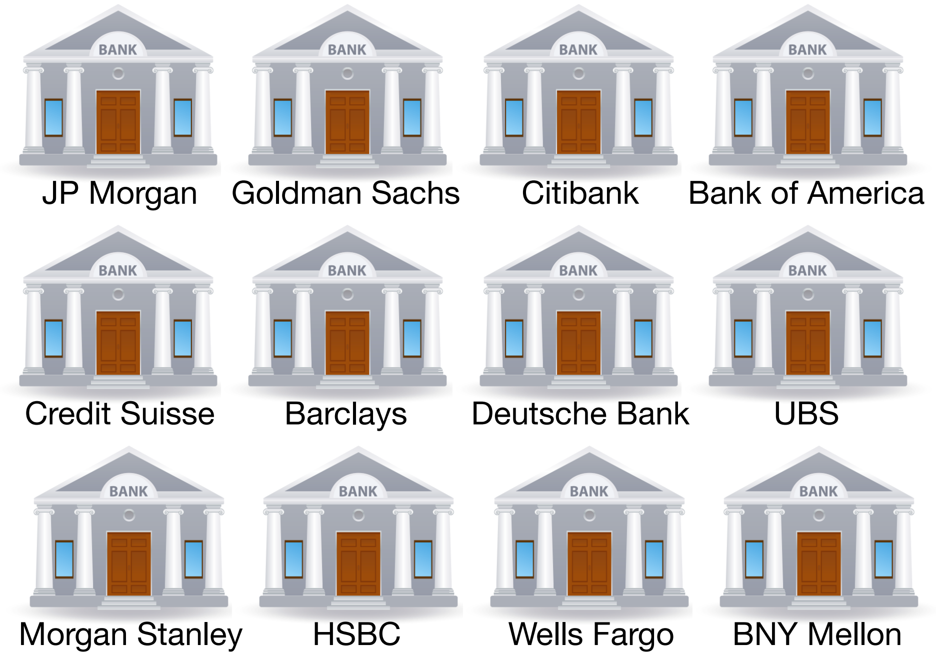
Application: zkLedger

- Privacy-preserving auditing for distributed ledgers
- A cryptographic system built out of:
  - Pedersen commitments and their homomorphism
  - Zero-knowledge proofs

First: the use case

(all cryptographic systems should have a use case)

Structure of the financial system



- Dozens of large investment banks
- Trading:
  - Securities
  - Currencies
  - Commodities
  - Derivatives
- Trillions of dollars

Financial Investments Regulatory Authority on OTC markets

A ledger records financial transactions

Assume a trusted ledger: append-only, immutable, consistent & visible to everyone

ID	Asset	From	To	Amount	
90	$	Citibank	Goldman Sachs	1,000,000	sig
91	€	JP Morgan	UBS	200,000	sig
92	€	JP Morgan	Barclays	3,000,000	sig

Citibank JP Morgan Barclays

Can verify important financial invariants

ID	Asset	From	To	Amount	
90	$	Citibank	Goldman Sachs	1,000,000	sig
91	€	JP Morgan	UBS	200,000	sig
92	€	JP Morgan	Barclays	3,000,000	sig

Verify

✓ Consent to transfer

Examining ledger:

✓ Has assets to transfer

✓ Assets neither created nor destroyed

Banks care about privacy

Trades reveal sensitive strategy information

Verifying invariants are maintained with privacy

ID	Asset	From	To	Amount	
90	$	Citibank	Goldman Sachs	1,000,000	sig
91	€	JP Morgan	UBS	200,000	sig
92	€	JP Morgan	Barclays	3,000,000	sig

Verify

Consent to transfer

Has assets to transfer

Assets neither created nor destroyed

Verifying invariants are maintained with privacy

ID	Asset	From, To, Amount
90	$	
91	€	
92	€	

Zerocash (zk-SNARKs) [S&P 2014]
Solidus (PVORM) [CCS 2017]

<u>Verify</u>

✓ Consent to transfer
✓ Has assets to transfer
✓ Assets neither created nor destroyed

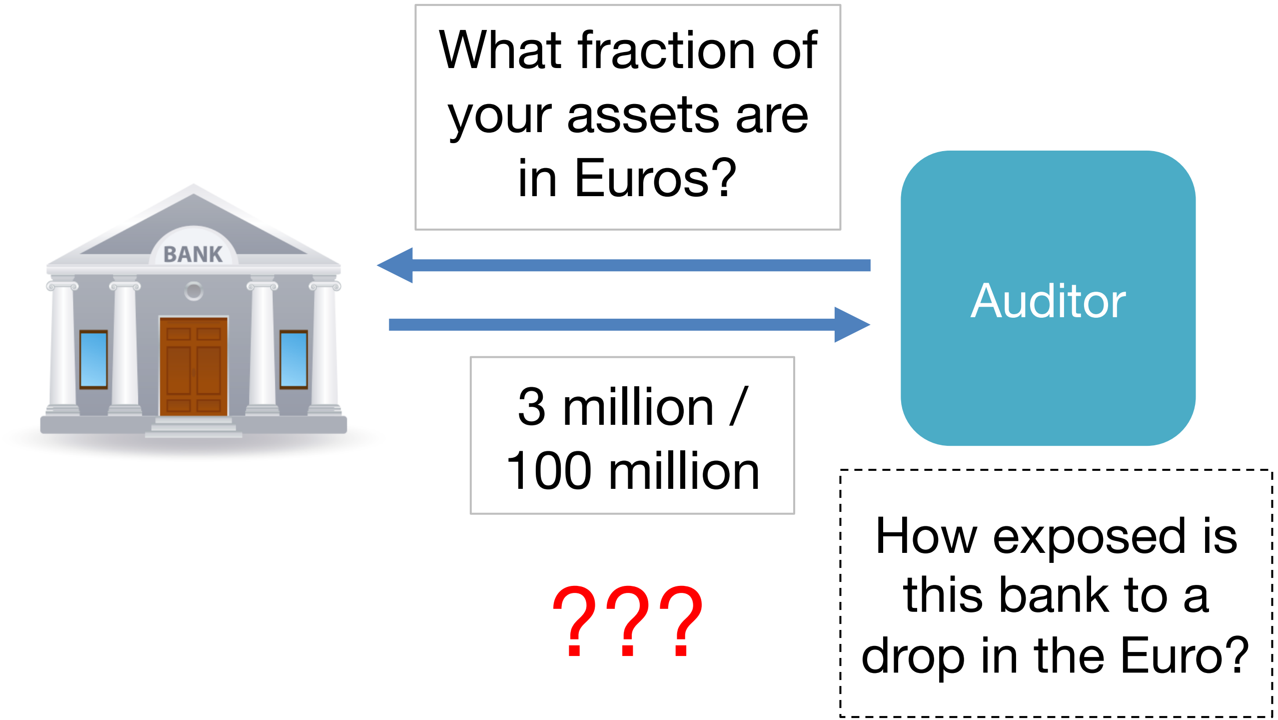
Problem

Regulators need insight into markets to maintain financial stability and protect investors

Participants would like to measure counterparty risk

- Leverage
- Exposure
- Overall market concentration

How to confidently audit banks to determine risk?



zkLedger

A private, auditable transaction ledger

- **Privacy:** Hides transacting banks and amounts
- **Integrity with public verification:** *Everyone* can verify transactions are well-formed
- **Auditing:** Compute provably-correct linear functions over transactions

Outline

- System & threat model
- zkLedger design
  - Pedersen commitments
  - Ledger table format
  - Zero-knowledge proofs
- Evaluation

Outline

- **System & threat model**
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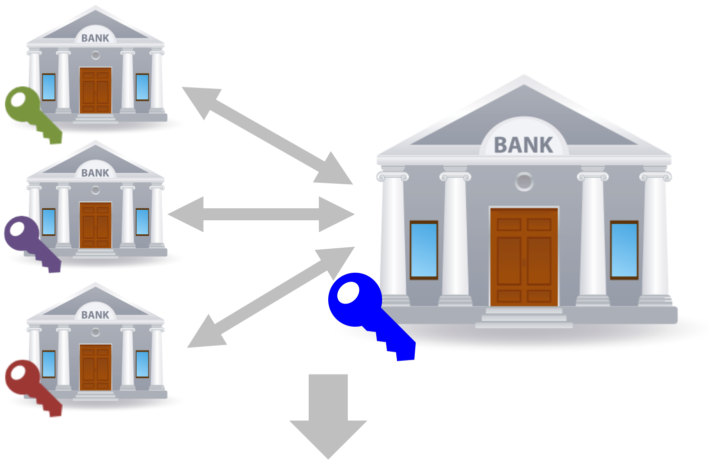
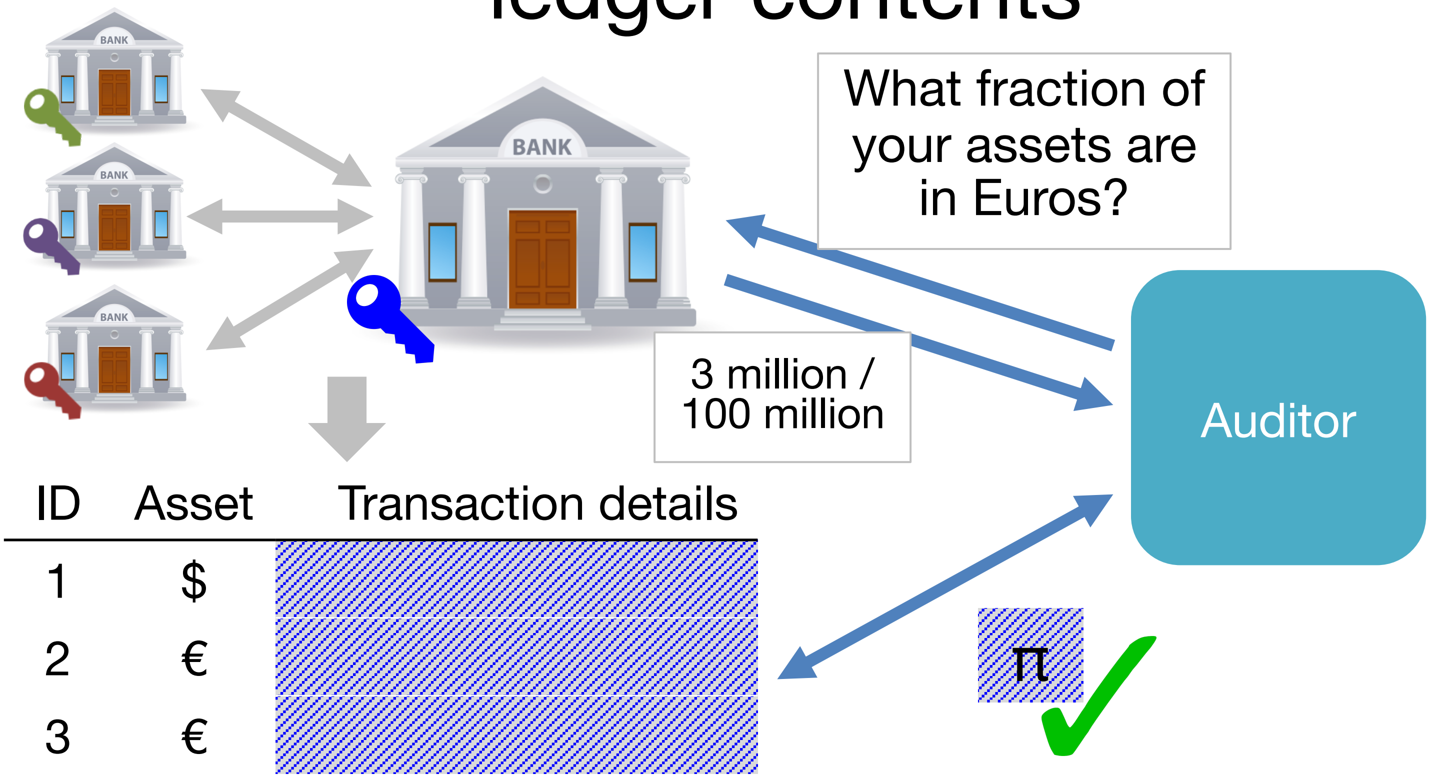
zkLedger system model



ID	Asset	Transaction details
1	$	
2	€	
3	€	

An auditor can obtain correct answers on ledger contents



ID	Asset	Transaction details
1	$	
2	€	
3	€	

Measurements zkLedger supports

- Ratios and percentages of holdings
- Sums, averages, variance, skew
- Outliers
- Approximations and orders of magnitude
- Changes over time
- Well-known financial risk measurements (Herfindahl-Hirschmann index)

Security goals

- The auditor and non-involved parties **cannot see** transaction participants or amounts
- Banks **cannot lie** to the auditor or **omit** transactions
- Banks **cannot violate** financial invariants
  - Honest banks can always **convince** the auditor of a correct answer
- A malicious bank **cannot block** other banks from transacting

Threat model

Banks might attempt to steal or hide assets, manipulate balances, or lie to the auditor

Banks can arbitrarily collude

Banks or the auditor might try to learn transaction contents

Out of scope:

- A ledger that omits transactions or is unavailable
- An adversary watching network traffic
- Banks leaking their own transactions

Outline

- System & threat model
- **zkLedger design**
  - Pedersen commitments
  - Ledger table format
  - Zero-knowledge proofs
- Evaluation

Example public transaction ledger

ID	Asset	From	To	Amount
1	€	Depositor	Goldman Sachs	30,000,000
2	€	Goldman Sachs	JP Morgan	10,000,000
3	€	JP Morgan	Barclays	1,000,000
4	€	JP Morgan	Barclays	2,000,000

Depositor injects assets to the ledger

ID	Asset	From	To	Amount
1	€	Depositor	Goldman Sachs	30,000,000
2	€	Goldman Sachs	JP Morgan	10,000,000
3	€	JP Morgan	Barclays	1,000,000
4	€	JP Morgan	Barclays	2,000,000

Goals: auditing + privacy

ID	Asset	From	To	Amount
1	€	Depositor	Goldman Sachs	30,000,000
2	€	Goldman Sachs	JP Morgan	10,000,000
3	€	JP Morgan	Barclays	1,000,000
4	€	JP Morgan	Barclays	2,000,000

Goals:

- Provably audit Barclays to find Euro holdings
- Hide participants, amounts, and transaction graph

Hide amounts with commitments

ID	Asset	From	To	Amount
1	€	Depositor	Goldman Sachs	30M
2	€	Goldman Sachs	JP Morgan	comm(10M)
3	€	JP Morgan	Barclays	comm(1M)
4	€	JP Morgan	Barclays	comm(2M)

comm(10M) × comm(1M) × comm(2M)

= comm(13M)

Hide participants with other techniques

ID	Asset	From	To	Amount
1	€	Depositor	Goldman Sachs	30M
2	€	Goldman Sachs	JP Morgan	`comm(10M)`
3	€	JP Morgan	Barclays	`comm(1M)`
4	€	JP Morgan	Barclays	`comm(2M)`

Strawman: audit by opening up combined commitments

ID	Asset	From	To	Amount
1	€	Depositor	Goldman Sachs	30M
2	€	Goldman Sachs	JP Morgan	`comm(10M)`
3	€	JP Morgan	Barclays	`comm(1M)`
4	€	JP Morgan	Barclays	`comm(2M)`

Reveals transactions

Barclays

How many Euros do you hold?

3 million

Auditor

Problems?

Open `comm(1M)` × `comm(2M)` to 3M

A malicious bank could omit transactions

ID	Asset	From	To	Amount
1	€	Depositor	Goldman Sachs	30M
2	€	Goldman Sachs	JP Morgan	`comm(10M)`
3	€	JP Morgan	Barclays	`comm(1M)`
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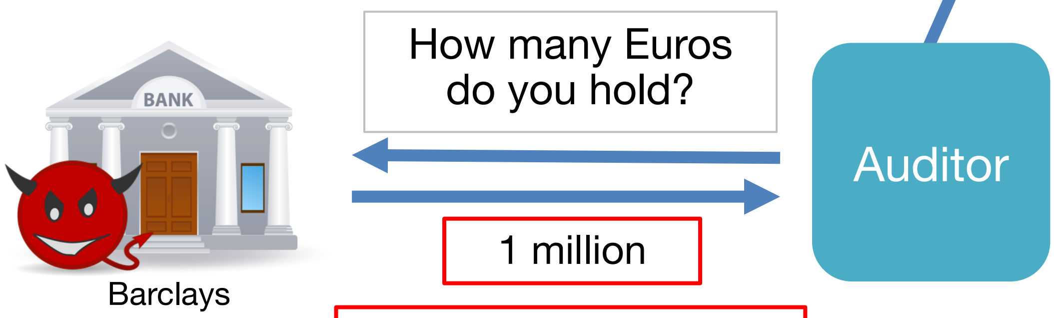


Open `comm(1M)` to 1M

A malicious bank could omit transactions

ID	Asset	From	To	Amount
1	€	Depositor	Goldman Sachs	30M
2	€	Goldman Sachs	JP Morgan	`comm(10M)`
3	€	JP Morgan	Barclays	`comm(1M)`
4	€	JP Morgan	Barclays	`comm(2M)`

zkLedger design: an entry for every bank in every transaction

ID	Asset	**Goldman Sachs**	**JP Morgan**	**Barclays**
1	€	Depositor, Goldman Sachs, 30M		
2	€	comm(-10M)	comm(10M)	**comm(0)**
3	€	**comm(0)**	comm(-1M)	comm(1M)
4	€	**comm(0)**	comm(-2M)	comm(2M)

Depositor transactions are public

Spender's column commits to negative value, receiver's positive value

For non-involved banks, entries commit to 0

Indistinguishable from commitments to non-zero values

Key insight: auditor audits *every* transaction

ID	Asset	Goldman Sachs	JP Morgan	Barclays
1	€	Depositor, Goldman Sachs, 30M		
2	€	`comm(-10M)`	`comm(10M)`	`comm(0)`
3	€	`comm(0)`	`comm(-1M)`	`comm(1M)`
4	€	`comm(0)`	`comm(-2M)`	`comm(2M)`

Barclays

How many Euros do you hold?

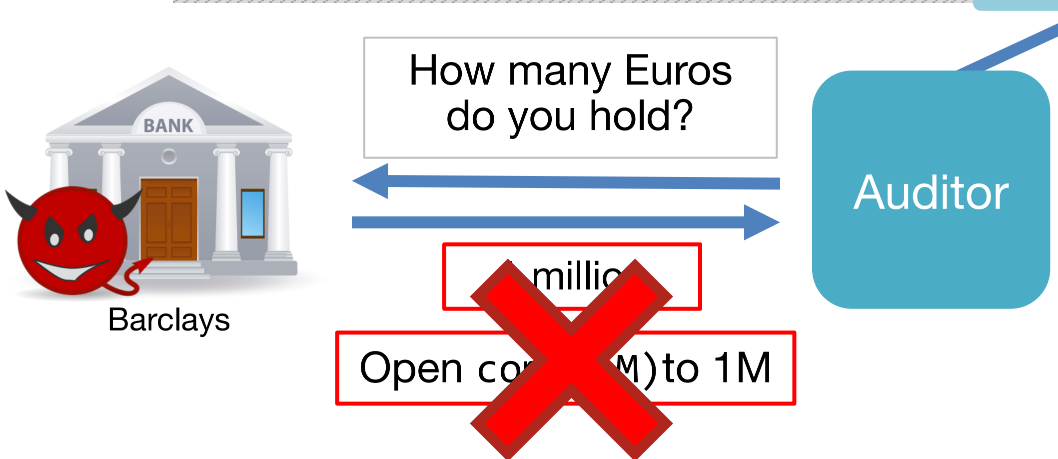
3 million

Auditor

Open [`comm(0)` × `comm(1M)` × `comm(2M)`] to 3M

A malicious bank can’t produce a proof for a different answer

ID	Asset	Goldman Sachs	JP Morgan	Barclays
1	€	Depositor, Goldman Sachs, 30M		
2	€	comm(-10M)	comm(10M)	comm(0)
3	€	comm(0)	comm(-1M)	comm(1M)
4	€	comm(0)	comm(-2M)	comm(2M)



Security goals

- ✓ The auditor and non-involved parties **cannot see** transaction participants, amounts, or transaction graph
- ✓ Banks **cannot lie** to the auditor or **omit** transactions
- Banks **cannot violate** financial invariants
  - Honest banks can always **convince** the auditor of a correct answer
- A malicious bank **cannot block** other banks from transacting

How to maintain financial invariants?

ID	Asset	Goldman Sachs	JP Morgan	Barclays	
1	€	Depositor, Goldman Sachs, 30M			
2	€	comm(-10M)	comm(10M)	comm(0)	comm(sig_{GS})
3	€	comm(0)	comm(-1M)	comm(1M)	comm(sig_{JP})
4	€	comm(0)	comm(-2M)	comm(2M)	comm(sig_{JP})

use non-interactive zero-knowledge proofs (NIZKs)!

What are the NIZK proof statements?

ID	Asset	Goldman Sachs	JP Morgan	Barclays	
1	€	Depositor, Goldman Sachs, 30M			
2	€	`comm(-10M)`	`comm(10M)`	`comm(0)`	comm(sig_{GS})
3	€	`comm(0)`	`comm(-1M)`	`comm(1M)`	comm(sig_{JP})
4	€	`comm(0)`	`comm(-2M)`	`comm(2M)`	comm(sig_{JP})

Sender proves in zero knowledge that it knows sk for signing, values committed to in row, and decommitment randomness for all of them such that :

- Values in the transaction row sum to zero
- Signature verifies with the PK of sending bank on that amount
- One bank receives, all others are zero
- Bank has assets to transfer from previous transactions

Preliminaries

- Anyone can compute the aggregate commitment for every bank i (over all transactions including this new transaction): $comm_{agg,i}$
- Let n be the number of banks
- $comm_{n+1}$ contains the signature on the transaction
- Let PK_i be the verification key of bank i with signing key SK_i
- Assume that the receiver obtains the decommitment values from the spender using an out-of-band channel

The spender proves in zero-knowledge that it knows

- s the index of spending bank, ℓ the index of receiving bank,
- decommitment values r_i and values v_i
- signature randomness r and sk,
- r_{agg}, v_{agg} for $comm_{agg,s}$,

such that:

- $comm_i$ opens up with r_i and v_i,
- v_{n+1} $is\ sig$ produced with r, sk and sig verifies with PK_s on transaction content

  [transaction is authorized]
- $v_s \leq 0, v_s = -v_\ell, \quad v_i = 0$ for $i \in [1, n] \neq \ell, s$,

  [spender loses money, receiver gains same money, the rest have zero]
- $comm_{agg,s}$ opens up with r_{agg} and v_{agg} and $v_{agg} \geq 0$

  [spender spends no more than resources]

Instead of one monolithic proof enforcing these properties, zkLedger does a set of more efficient things but they are less relevant here

Outline

- System model
- zkLedger design
  - Hiding commitments
  - Ledger table format
  - Zero-knowledge proofs
- **Evaluation**

Implementation

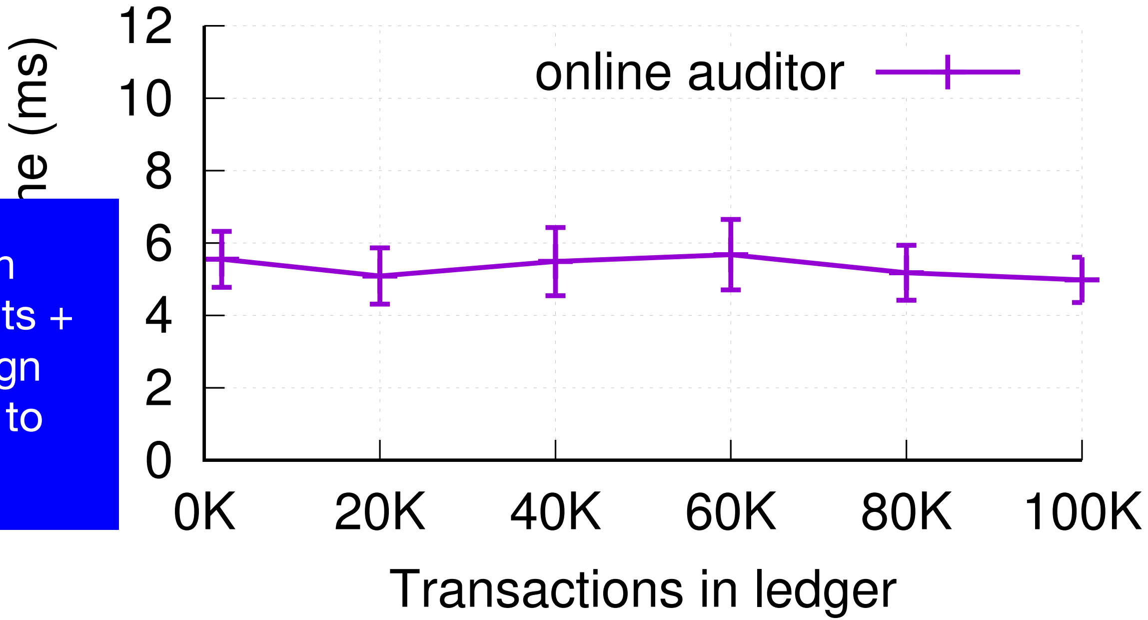
- zkLedger written in Go
- Elliptic curve library: btcec, secp256k1
- ~4000 loc

Evaluation

- How fast is auditing?
- How does zkLedger scale with the number of banks?

Experiments on 12 4 core Intel Xeon 2.5Ghz VMs, 24 GB RAM

Simple auditing is fast and independent of ledger size



Pedersen commitments + table design amenable to caching

Auditing 4 banks measuring market concentration

Cost in a transaction per bank

- Entry size: **4.5KB**
- Creating an entry: **8ms**
- Verifying an entry: **7ms**

× # banks

Highly parallelizable

Significant opportunities for compression and speedup

Summary

- Specialized/partial homomorphic encryption enables specific functionalities and tend to be faster than FHE at computing these
- Pedersen commitment is also homomorphic
- zkLedger provides privacy and auditing on transaction ledgers using Pedersen commitments, their homomorphism and NIZKs